Supplementary appendix: Optimal margins and equilibrium prices

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Abstract

In this supplementary appendix we study the case in which investors, who may face margin calls in the future, can keep a fraction of their initial endowment as cash. They invest the rest of their endowment in the risky asset. Holding cash avoids the sale of assets at fire-sale prices to satisfy margin calls. We derive a condition under which investors find it optimal to invest their entire initial endowment in the risky asset and do not invest in cash.

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1 Adding cash to the model

Compared to our baseline model, we now allow investors to keep a fraction of their initial endowment in cash. We let β denote the fraction of the endowment kept in cash at $t = 0$. The remaining fraction $1-\beta$ is invested in the risky asset. Cash holdings are observable and do not require costly risk-management effort. Hence, there is no moral hazard associated with investors' cash holdings. Feasibility requires

$$0 \leq \beta \leq 1. \tag{1}$$

Investors' transfers are constrained by limited liability. An investor cannot make transfers larger than what is returned by cash holdings β, the fraction $(1-\beta)(1-\alpha(s))$ of assets under her management and by the fraction $(1-\beta)\,\alpha(s)$ of assets she deposited on the margin account. Thus,

$$\tau(\theta, s, R) \leq \beta + (1-\beta)\left[\alpha(s)p + (1-\alpha(s))R\right], \quad \forall(\theta, s, R). \tag{2}$$

With cash, a sophisticated investor's incentive constraint for risk-prevention effort is

$$\beta + (1-\beta)\left(\alpha(s)p(s) + (1-\alpha(s))\mathcal{P}\right) \geq E[\tau(\tilde{\theta}, \tilde{s})|\tilde{s} = s]. \tag{3}$$

The pledgeable return of cash is equal to its physical return of one. Cash holdings do not depend on the realization of the signal $\tilde{s}$ at $t = 1$ because the decision to hold cash occurs at $t = 0$, before the realization of the signal $\tilde{s}$.[1]

With cash, the participation constraint of a sophisticated investor is

$$-E[\tau(\tilde{\theta}, \tilde{s})] \geq E\left[\beta\,(R - C - 1) + (1-\beta)\,\alpha(\tilde{s})(R - C - p(\tilde{s}))\right], \tag{4}$$

as the opportunity cost of using cash is $R - C - 1 > 0$.[2]

2 Optimal contract under moral hazard

In this section, we derive the privately optimal contract between the hedger and the sophisticated investor, taking the price p as given. The next proposition characterizes basic properties of the optimal contract under moral hazard.

[1] As in the case without cash, the derivation of the incentive constraint takes into account the maximal relaxation of the constraint in case of investor default when all remaining resources are transferred to hedgers, $\tau(\tilde{\theta}, \tilde{s}, 0) = \beta + \alpha(s)p(s)$.

[2] It follows that cash is not used in the first-best as cash holdings offer no benefit but entail an opportunity cost.

Proposition 1 *The incentive constraint after a bad signal and the participation constraint are binding. The incentive constraint after a good signal as well as the limited liability constraints in states* $(\bar{\theta}, \bar{s})$ *and* $(\bar{\theta}, \underline{s})$ *are slack. Margins are not used after a good signal,* $\alpha(\bar{s}) = 0$*.*

The results in Proposition 1 are similar to those obtained in the case where the sophisticated investor is assumed not to hold cash. When all limited liability constraints are slack, the expected transfers conditional on the signal (as a function of β, $\alpha(\underline{s})$, and p) are given by

$$E[\tau(\tilde{\theta}, \tilde{s})|\tilde{s} = \underline{s}] = \beta + (1 - \beta)\left(\alpha(\underline{s})p + (1 - \alpha(\underline{s}))\mathcal{P}\right) > 0 \tag{5}$$

and

$$\begin{aligned} E[\tau(\tilde{\theta}, \tilde{s})|\tilde{s} = \bar{s}] = &- \frac{\beta\left(R - C - 1 + \text{prob}[\underline{s}]\right)}{\text{prob}[\bar{s}]} \\ &- \frac{\text{prob}[\underline{s}]\left(1 - \beta\right)}{\text{prob}[\bar{s}]}\left[\alpha(\underline{s})\left(R - C\right) + (1 - \alpha(\underline{s}))\mathcal{P}\right] < 0, \end{aligned} \tag{6}$$

where we dropped the reference to the signal s in the price p because margins are not used after a good signal.

Thus, conditional on the realization of the signal $\tilde{s}$, the optimal contract provides full insurance and we can write a hedger's consumption after a bad and a good signal as

$$\underline{c} = E[\theta|\underline{s}] + \beta + (1 - \beta)\left(\alpha(\underline{s})p + (1 - \alpha(\underline{s}))\mathcal{P}\right) \tag{7}$$

and

$$\bar{c} = E[\theta|\bar{s}] - \frac{\beta\left(R - C - 1 + \text{prob}[\underline{s}]\right)}{\text{prob}[\bar{s}]} - \frac{\text{prob}[\underline{s}]\left(1 - \beta\right)\left[\alpha(\underline{s})\left(R - C\right) + (1 - \alpha(\underline{s}))\mathcal{P}\right]}{\text{prob}[\bar{s}]}. \tag{8}$$

The expected utility of a hedger is

$$\text{prob}[\bar{s}]u(\bar{c}) + \text{prob}[\underline{s}]u(\underline{c}),$$

where $\underline{c}$ and $\bar{c}$ are given above in (7) and (8). The derivative of the hedger's expected utility with respect to α is (weakly) positive if

$$\varphi(\alpha(\underline{s}), p) \equiv \frac{u'(\underline{c})}{u'(\bar{c})} \geq -\frac{\text{prob}[\bar{s}]\,\frac{d\bar{c}}{d\alpha}}{\text{prob}[\underline{s}]\,\frac{d\underline{c}}{d\alpha}} = 1 + \frac{R - C - p}{p - \mathcal{P}}. \tag{9}$$

The numerator of the fraction in the last term on the right-hand side is the opportunity cost of margins, the denominator is the incentive benefit of margins. The derivative of the hedger's expected utility with respect to β is (weakly) positive if

$$\varphi(\alpha(\underline{s}), p) \equiv \frac{u'(\underline{c})}{u'(\bar{c})} \geq -\frac{\text{prob}[\bar{s}]}{\text{prob}[\underline{s}]} \frac{\frac{d\bar{c}}{d\beta}}{\frac{d\underline{c}}{d\beta}} = 1 + \frac{(R-C-1) - \text{prob}[\underline{s}]\alpha(\underline{s})(R-C-p)}{\text{prob}[\underline{s}](1-\mathcal{P}-\alpha(\underline{s})(p-\mathcal{P}))}. \tag{10}$$

The numerator of the fraction in the last term on the right-hand side is the opportunity cost of cash holdings relative to the opportunity cost of margins. The denominator is the incentive benefit of cash relative to that of margins.

An interior solution, $(\alpha, \beta) \in (0,1)^2$, would require that (9) and (10) hold as equalities. This is generically not the case, however, because the right-hand sides of (9) and (10) are generically not equal. To see why, note the following: Because the consumptions in (7) and (8) are linear in α and β, the right-hand side of (9) is independent of α and that of (10) is independent of β. Furthermore, because in (7) and (8) the terms involving α are proportional to $(1-\beta)$, the right-hand side of (9) is also independent of β. Finally, because the ratio of the slopes in the hyperbola in $\frac{\frac{d\bar{c}}{d\beta}}{\frac{d\underline{c}}{d\beta}}$ is equal to the fraction in $\frac{\frac{d\bar{c}}{d\alpha}}{\frac{d\underline{c}}{d\alpha}}$, α cancels when one equates the right-hand side of (9) to that of (10).

This mathematical result reflects the economic fact that α and β are substitutes: Both instruments serve the same purpose (relaxing the incentive constraint) and have the same type of drawback (the opportunity cost of not investing optimally). Either α or β has the more attractive ratio of costs to benefits, and the instrument with the worse ratio of the two is never used in the optimal contract. This is stated in the next proposition.

Proposition 2 *For $\mathcal{P} \geq 1 (> p)$, neither cash nor margins are used, $\alpha^* = \beta^* = 0$. For $\mathcal{P} < 1$, the following holds. When*

$$prob[\underline{s}]\frac{R-C-p}{p-\mathcal{P}} \leq \frac{R-C-1}{1-\mathcal{P}} \tag{11}$$

holds, cash is not used, $\beta^* = 0$. When condition (11) is reversed margins are not used, $\alpha^* = 0$. When (11) holds as an equality, the optimal contract is indifferent between cash and margins.

Inequality (11) states that the ratio of opportunity costs to incentive benefits is better for margins than for cash. Thus, when (11) holds, cash holdings are never used, while in the opposite case margins are never used. Inequality (11) can be rewritten as

$$p \geq \frac{\mathcal{P}(R-C-1)+\text{prob}[\underline{s}](R-C)(1-\mathcal{P})}{(R-C-1)+\text{prob}[\underline{s}](1-\mathcal{P})} \in (\mathcal{P},1), \tag{12}$$

which shows that if p (the price at which assets are liquidated following the margin call) is high enough, margins offer a more attractive cost-benefit ratio than cash holdings.

3 Zero cash holdings in the market equilibrium

We can now characterize the set of parameters for which cash is not used in the market equilibrium.

Proposition 3 *If*

$$\delta \leq \frac{(R-C-1)\,(R-C-\mathcal{P})}{R-C-1+\textit{prob}[\underline{s}]\,(1-\mathcal{P})} \tag{13}$$

holds, optimal cash holdings in the market equilibrium are zero, $\beta^* = 0$.

Inequality (13) states that δ is not too large, i.e., the inefficiency generating by reallocating the assets to the unsophisticated investors is not too large. Correspondingly, the unsophisticated investors buy the assets at a relatively high price. Hence, the cost-benefit ratio of margins is attractive, and margins are used instead of cash holdings.

Proofs

Proof of Proposition 1 Form the Lagrangian using the objective

$$E[u(\tilde{\theta}+\tau(\tilde{\theta},\tilde{s},\tilde{R}))], \tag{14}$$

the limited liability constraints (2) with multipliers $\mu_{LL}(\theta,s)$, the feasibility constraints on margins ($0 \leq \alpha \leq 1$) with $\mu_0(s)$ for $\alpha(s) \geq 0$ and $\mu_1(s)$ for $\alpha(s) \leq 1$, the feasibility constraints on cash holdings (1) with μ_0^β for $\beta \geq 0$ and μ_1^β for $\beta \leq 1$, the participation constraint (4) with multiplier μ and the incentive compatibility constraints (3) with multipliers $\mu_{IC}(s)$.

The first-order conditions of the Lagrangian with respect to $\tau(\theta,s)$ are

$$\text{prob}[\theta,s]u'(\theta+\tau(\theta,s)) - \mu\text{prob}[\theta,s] + \mu_{LL}(\theta,s) + \text{prob}[\theta|s]\mu_{IC}(s) = 0 \qquad \forall(\theta,s). \tag{15}$$

Rearranging, we obtain

$$u'(\theta+\tau(\theta,s)) = \mu + \frac{\mu_{LL}(\theta,s)}{\text{prob}[\theta,s]} + \frac{\mu_{IC}(s)}{\text{prob}[s]} \qquad \forall(\theta,s) \tag{16}$$

where we used that $\text{prob}[\theta|s]\text{prob}[s] = \text{prob}[\theta, s]$.

We conjecture, and verify later, that the limited liability constraints in $(\bar{\theta}, s)$ states are always slack. That is, $\mu_{LL}(\bar{\theta}, s) = 0$. As for the limited liability constraints in $(\underline{\theta}, s)$ states, in what follows we focus on the set of parameters under these constraints are slack.

We now show by contradiction that the participation constraint (4) binds. Suppose not. Plugging $\mu = 0$ and $\mu_{LL}(\bar{\theta}, s) = 0$ into (16) implies that $\mu_{IC}(s) > 0$ for all s. Hence, both incentive constraints bind, $E[\tau(\tilde{\theta}, \tilde{s})|\tilde{s} = s] = \beta + (1-\beta)(\alpha(s)p(s) + (1-\alpha(s))\mathcal{P})$ for $s = \bar{s}, \underline{s}$. Therefore,

$$E[\tau(\tilde{\theta}, \tilde{s})] = E[E[\tau(\tilde{\theta}, \tilde{s})|\tilde{s}]] = E[\beta + (1-\beta)(\alpha(\tilde{s})p(\tilde{s}) + (1-\alpha(\tilde{s}))\mathcal{P})] \tag{17}$$

From the participation constraint, we have

$$\begin{aligned} 0 &\le -E[\tau(\tilde{\theta}, \tilde{s})] - \beta(R - C - 1) - (1-\beta) E[\alpha(\tilde{s})(R - C - p(\tilde{s}))] \\ &= -E[\beta + (1-\beta)(\alpha(\tilde{s})p(\tilde{s}) + (1-\alpha(\tilde{s}))\mathcal{P})] - \beta(R - C - 1) - (1-\beta) E[\alpha(\tilde{s})(R - C - p(\tilde{s}))] \\ &= -E[\beta(R - C) + (1-\beta)\alpha(\tilde{s})(R - C) + (1-\beta)(1-\alpha(\tilde{s}))\mathcal{P}], \end{aligned}$$

where we used (17) to go from the first to the second line. The expression on the right-hand side of the last line is strictly negative since $R - C > \mathcal{P} > 0$, $0 \le \beta \le 1$, and $0 \le \alpha(\tilde{s}) \le 1$. A contradiction. Hence, $\mu > 0$ and the participation constraint binds.

We now show that the incentive constraint after a good signal is slack while the incentive constraint after a bad signal is binding. First note that it cannot be that both incentive constraints are slack since we assume that the first-best is not attainable, $\mathcal{P} < (\pi - \underline{\pi})\Delta\theta$. It also cannot be that both incentive constraints bind (see the argument showing that the participation constraint binds above).

We now show by contradiction that the incentive constraint following a bad signal binds. Suppose not and hence $\mu_{IC}(\underline{s}) = 0$. Since we are considering a set of parameters under which the limited liability constraints are slack, we have by (16) that

$$\begin{aligned} u'(\theta + \tau(\theta, \bar{s})) &= \mu + \frac{\mu_{IC}(\bar{s})}{\text{prob}[\bar{s}]} \\ u'(\theta + \tau(\theta, \underline{s})) &= \mu \end{aligned}$$

so that there is full risk-sharing conditional on the signal and hence

$$\tau(\underline{\theta}, s) - \tau(\bar{\theta}, s) = \Delta\theta > 0 \quad \forall s. \tag{18}$$

Moreover, since $\mu_{IC}(\bar{s}) \geq 0$, it follows that $u'(\theta + \tau(\theta, \underline{s})) \leq u'(\theta + \tau(\theta, \bar{s}))$, and thus

$$\tau(\theta, \underline{s}) \geq \tau(\theta, \bar{s}) \qquad \forall \theta. \tag{19}$$

From the binding participation constraint

$$-\left[\text{prob}[\bar{s}]E[\tau(\tilde{\theta}, \tilde{s})|\tilde{s} = \bar{s}] + \text{prob}[\underline{s}]E[\tau(\tilde{\theta}, \tilde{s})|\tilde{s} = \underline{s}]\right] = \\ E\left[\beta\left(R - C - 1\right) + \left(1 - \beta\right)\alpha(\tilde{s})(R - C - p(\tilde{s}))\right].$$

Since the right-hand side is non-negative, we know that

$$-\left[\text{prob}[\bar{s}]E[\tau(\tilde{\theta}, \tilde{s})|\tilde{s} = \bar{s}] + \text{prob}[\underline{s}]E[\tau(\tilde{\theta}, \tilde{s})|\tilde{s} = \underline{s}]\right] \geq 0$$

while $E[\tau(\tilde{\theta}, \tilde{s})|\tilde{s} = \bar{s}] > 0$ (binding incentive constraint after a good signal). This implies that

$$E[\tau(\tilde{\theta}, \tilde{s})|\tilde{s} = \underline{s}] < 0 \tag{20}$$

Using (18), (19) and $\bar{\pi} > \underline{\pi}$, we can write

$$\begin{aligned} 0 &< E[\tau(\tilde{\theta}, \tilde{s})|\tilde{s} = \bar{s}] \equiv \bar{\pi}\tau(\bar{\theta}, \bar{s}) + (1 - \bar{\pi})\tau(\underline{\theta}, \bar{s}) \\ &< \underline{\pi}\tau(\bar{\theta}, \bar{s}) + (1 - \underline{\pi})\tau(\underline{\theta}, \bar{s}) \\ &\leq \underline{\pi}\tau(\bar{\theta}, \underline{s}) + (1 - \underline{\pi})\tau(\underline{\theta}, \underline{s}) \equiv E[\tau(\tilde{\theta}, \tilde{s})|\tilde{s} = \underline{s}] \end{aligned}$$

But this contradicts (20). Thus, the incentive constraint after a good signal is slack while the incentive constraint after a bad signal binds.

We now show that there is no margin call after a good signal, $\alpha(\bar{s}) = 0$. Since we are considering a set of parameters under which the limited liability constraints are slack, the first-order conditions of the Lagrangian with respect to $\alpha(s)$ write as

$$\mu_0(s) - \mu_1(s) + (1 - \beta)\,\mu_{IC}(s)(p(s) - \mathcal{P}) = \mu\,(1 - \beta)\,\text{prob}[s]\,(R - C - p(s)) \qquad \forall s. \tag{21}$$

Because the incentive constraint after a good signal is slack, we have $\mu_{IC}(\bar{s}) = 0$. If $\beta < 1$, then $\mu_0(\bar{s}) - \mu_1(\bar{s}) > 0$, which implies $\alpha(\bar{s}) = 0$. If $\beta = 1$, then we can set $\alpha(\bar{s}) = 0$ without loss of generality.

When limited liability constraints are slack, there is full-risk-sharing conditional on the realization of the signal $\tilde{s}$:

$$u'(\theta + \tau(\theta, \tilde{s})) = \mu + \frac{\mu_{IC}(\tilde{s})}{\text{prob}[\tilde{s}]} \tag{22}$$

so that (18) holds.

Combining the binding participation constraint and the incentive constraint after a bad signal with (18) gives the following characterization of the optimal transfers:

$$\begin{aligned}\tau(\bar{\theta},\bar{s}) =& -(1-\bar{\pi})\Delta\theta - \frac{\beta\,(R-C-1+\text{prob}(\underline{s}))}{\text{prob}[\bar{s}]} \\ &- \frac{\text{prob}[\underline{s}]\,(1-\beta)\,[\alpha(\underline{s})\,(R-C)+(1-\alpha(\underline{s}))\mathcal{P}]}{\text{prob}[\bar{s}]}\end{aligned} \tag{23}$$

$$\begin{aligned}\tau(\underline{\theta},\bar{s}) =& \bar{\pi}\Delta\theta - \frac{\beta\,(R-C-1+\text{prob}(\underline{s}))}{\text{prob}[\bar{s}]} \\ &- \frac{\text{prob}[\underline{s}]\,(1-\beta)\,[\alpha(\underline{s})\,(R-C)+(1-\alpha(\underline{s}))\mathcal{P}]}{\text{prob}[\bar{s}]}\end{aligned} \tag{24}$$

$$\tau(\bar{\theta},\underline{s}) = -(1-\underline{\pi})\Delta\theta + \beta + (1-\beta)\,(\alpha(\underline{s})p + (1-\alpha(\underline{s}))\mathcal{P}) \tag{25}$$

$$\tau(\underline{\theta},\underline{s}) = \underline{\pi}\Delta\theta + \beta + (1-\beta)\,(\alpha(\underline{s})p + (1-\alpha(\underline{s}))\mathcal{P}) \tag{26}$$

It is immediate that the limited liability constraint in state $(\bar{\theta},\bar{s})$ is slack because $\tau(\bar{\theta},\bar{s}) < 0$ (as $R-C > 1$). To show that the limited liability constraint in state $(\bar{\theta},\underline{s})$ is slack, we substitute $\tau(\bar{\theta},\underline{s})$ into the limited liability constraint, which then simplifies to

$$-(1-\underline{\pi})\Delta\theta \leq (1-\beta)(1-\alpha)(R-\mathcal{P}),$$

which always holds. QED

Proof of Proposition 2 Considering again the set of parameters such that limited liability constraint are slack, the first-order conditions of the Lagrangian with respect to β and $\alpha(\underline{s})$ are

$$\mu_0^\beta - \mu_1^\beta + \mu_{IC}[1-(\alpha p + (1-\alpha)\mathcal{P})] = \mu\big[(R-C-1) - \text{prob}[\underline{s}]\alpha(R-C-p)\big] \tag{27}$$

$$\mu_0 - \mu_1 + (1-\beta)\,\mu_{IC}(p-\mathcal{P}) = \mu\,(1-\beta)\,\text{prob}[s]\,(R-C-p) \tag{28}$$

where we dropped the reference to $\underline{s}$ in p, α and the Lagrange multipliers.

If $\beta = 1$, then we can set $\alpha(\underline{s}) = 0$ without loss of generality (as we want to show that either cash or margins are used). So we proceed with $\beta < 1$ (and hence $\mu_1^\beta = 0$).

Next, as we only consider parameters for which all limited liability constraints are slack, it must be that $\alpha < 1$. Suppose not, $\alpha = 1$. Then, using (26), $\tau(\underline{\theta},\underline{s}) = \underline{\pi}\Delta\theta + \beta + (1-\beta)p > \beta + (1-\beta)p$. But this violates the limited liability constraint.

We now show that when $\mathcal{P} \geq p$, then margins are not used, $\alpha(\underline{s}) = 0$. For $\beta = 1$, we already know that margins are not used. Consider $\beta < 1$. The right-hand side of (28) is strictly positive since $R - C > 1 > p$, $\mu > 0$ and $\beta < 1$. If $\mathcal{P} \geq p$, then $\mu_0 > 0$ must hold and $\alpha(\underline{s}) = 0$.

Next, we show that when pledgeable income is high enough, $\mathcal{P} > 1$, cash is not used. First note that if $\mathcal{P} > 1$, then $\mathcal{P} \geq p$ and $\alpha(\underline{s}) = 0$. Then, (27) simplifies to

$$\mu_0^\beta - \mu_1^\beta + \mu_{IC}(1 - \mathcal{P}) = \mu(R - C - 1). \tag{29}$$

The right-hand side of (29) is positive since $R - C > 1$ and $\mu > 0$. On the left-hand side, as $1 - \mathcal{P} < 0$ we then have $\mu_0^\beta > 0$ and hence, $\beta = 0$.

It remains to characterize what happens when $\mathcal{P} < p < 1$. Since $\mu > 0$, $\alpha < 1$, $\beta < 1$ and $\mathcal{P} < p < 1$, we can write (27) and (28) as

$$\frac{\mu_{IC}}{\text{prob}[\underline{s}]\mu} = \frac{(R - C - 1) - \text{prob}[\underline{s}]\alpha(R - C - p)}{\text{prob}[\underline{s}][1 - (\alpha p + (1 - \alpha)\mathcal{P})]} - \frac{\mu_0^\beta}{\text{prob}[\underline{s}]\mu[1 - (\alpha p + (1 - \alpha)\mathcal{P})]} \tag{30}$$

$$\frac{\mu_{IC}}{\text{prob}[\underline{s}]\mu} = \frac{R - C - p}{p - \mathcal{P}} - \frac{\mu_0}{\text{prob}[\underline{s}]\mu(1 - \beta)(p - \mathcal{P})} \tag{31}$$

and hence

$$\frac{(R - C - 1) - \text{prob}[\underline{s}]\alpha(R - C - p)}{\text{prob}[\underline{s}][1 - (\alpha p + (1 - \alpha)\mathcal{P})]} - \hat{\mu}_0^\beta = \frac{R - C - p}{p - \mathcal{P}} - \hat{\mu}_0 \tag{32}$$

where $\hat{\mu}_0 = \frac{\mu_0}{\text{prob}[\underline{s}]\mu(1-\beta)(p-\mathcal{P})}$ and $\hat{\mu}_0^\beta = \frac{\mu_0^\beta}{\text{prob}[\underline{s}]\mu[1-(\alpha p+(1-\alpha)\mathcal{P})]}$. After some manipulation, (32) can be written as

$$\frac{(R - C - 1)(p - \mathcal{P}) - \text{prob}[\underline{s}](R - C - p)(1 - \mathcal{P})}{\text{prob}[\underline{s}](p - \mathcal{P})[1 - (\alpha p + (1 - \alpha)\mathcal{P})]} = \hat{\mu}_0^\beta - \hat{\mu}_0. \tag{33}$$

First, note that the denominator of (33) is strictly positive. So if

$$\frac{R - C - 1}{1 - \mathcal{P}} > \text{prob}[\underline{s}]\frac{R - C - p}{p - \mathcal{P}} \tag{34}$$

the left-hand side of (33) is strictly positive. Then $\hat{\mu}_0^\beta > 0$ (as $\hat{\mu}_0 \geq 0$) and hence, $\beta = 0$.

Similarly, when the inequality in (33) is reverse, we have $\alpha = 0$. Finally, when (33) holds as an equality, then there is no difference between cash and margins. QED

Proof of Proposition 3 Suppose, contrary to the claim that (13) holds and yet $\beta^* > 0$. Condition (13) is equivalent to

$$\frac{\mathcal{P}(R - C - 1) + \text{prob}[\underline{s}](R - C)(1 - \mathcal{P})}{(R - C - 1) + \text{prob}[\underline{s}](1 - \mathcal{P})} \leq R - C - \delta. \tag{35}$$

When $\beta^* > 0$, then $\alpha^* = 0$ and the market clearing price p must be such that (Proposition 2)

$$p < \frac{\mathcal{P}(R - C - 1) + \text{prob}[\underline{s}](R - C)(1 - \mathcal{P})}{(R - C - 1) + \text{prob}[\underline{s}](1 - \mathcal{P})}. \tag{36}$$

Since the supply of assets is 0, the market must clear with 0 trade. This implies the price must be such that

$$R - C - \delta \leq p. \tag{37}$$

Together, (35), (36) and (37) imply

$$p < \frac{\mathcal{P}(R - C - 1) + \text{prob}[\underline{s}](R - C)(1 - \mathcal{P})}{(R - C - 1) + \text{prob}[\underline{s}](1 - \mathcal{P})} \leq R - C - \delta < p,$$

a contradiction. QED